Zuweisungsregel

$$\left\{P_E^x\right\} x := E \left\{P\right\}$$

Sequenzregel

$$\frac{\left\{P\right\} S_1 \left\{Q\right\}; \left\{Q\right\} S_2 \left\{R\right\}}{\left\{P\right\} S_1; S_2 \left\{R\right\}}$$

Iteration

$$\frac{\left\{P \wedge B\right\} S \left\{P\right\}}{\left\{P\right\} \text{while } B \text{ do } S \left\{P \wedge \neg B\right\}}$$

$$\frac{\left\{P\right\} S \left\{Q\right\} \qquad \left\{Q \wedge \neg B\right\} S \left\{Q\right\}}{\left\{P\right\} \text{repeat } S \text{ until } B \left\{Q \wedge B\right\}}$$

Fallunterscheidung

$$\frac{\{P \wedge B\}\, S\, \{Q\} \qquad P \wedge \neg B \Rightarrow Q}{\{P\}\ \text{if } B \text{ then } S\ \{Q\}}$$

$$\frac{\{P \wedge B\}\, S_1\, \{Q\} \qquad \{P \wedge \neg B\}\, S_2\, \{Q\}}{\{P\}\ \text{if } B \text{ then } S_1 \text{ else } S_2\ \{Q\}}$$

Implikation

$$\frac{\{P\}\, S\, \{Q\}, Q \Rightarrow R}{\{P\}\, S\, \{R\}}$$

$$\frac{P \Rightarrow R, \{R\}\, S\, \{Q\}}{\{P\}\, S\, \{Q\}}$$

Aufgabe 1

$\{\textit{weakest precondition}\} \equiv P$

x = x*20

x= x*x+y

$\{x < 1000\}$

$\{x < 1000\}$

x= x*x+y

$\{x * x + y < 1000\}$

x = x*20

$\{(x * 20) * (x * 20) + y < 1000\}$

$P \equiv \{400x^2 + y < 1000\}$

$R \equiv \{400x^2 + y < 100\}$ stärker!

$R \Rightarrow P$ R ist stärker als P

$R \Rightarrow P \wedge \{P\}C\{Q\}$ P ist schwächere Vorbedingung

Gesucht: schwächste (am wenigsten einschränkende) Vorbedingung:

$wp(C,Q)$

Analog: stärkste (trennschärfste) Nachbedingung

Aufgabe 2

$\{x > 0 \wedge y > 0\}$

x = x+5

x= x*y

$\{strongest\ postcondition\} \equiv Q$

Lösung

$\{x > 0 \wedge y > 0\} \equiv$

$\{x + 5 > 5 \wedge y > 0\}$

x= x+5

$\{x > 5 \wedge y > 0\} \equiv$

$\{x * y > 5 * y \wedge y > 0\}$

x= x*y

$\{x > 5 * y \wedge y > 0\} \equiv$

$\{x / y - 5 > 0 \wedge y > 0\}$

stärker $\{x > 6 * y \wedge y > 0\}$

aber keine Nachbedingung

schwächer $\{x / y > 0 \wedge y > 0\}$

Aufgabe 3

$\{x \geq 0\}$

a=0

b=1

c=1

$\{a^2 \leq x \wedge b = (a+1)^2 \wedge c = 2a+1 \wedge a \geq 0\}$

ahile (b<=x)

a=a+1

c=c+2

b=b+c

$\{a^2 \leq x < (a+1)^2\}$

Aufgabe 3

$\{x \geq 0\}$

a=0
b=1
c=1

$\{a^2 \leq x \wedge b = (a+1)^2 \wedge c = 2a+1 \wedge a \geq 0\}$

Einsetzen, ***nachrechnen, stimmt!

ahile (b<=x)
    a=a+1
    c=c+2
    b=b+c

$\{a^2 \leq x < (a+1)^2\}$

$$\frac{\{P \wedge B\}\, S\, \{P\}}{\{P\} \text{ while } B \text{ do } S \{P \wedge \neg B\}}$$

Aufgabe 3

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$\{a^2 \leq x \wedge b = (a+1)^2 \wedge c = 2a+1 \wedge a \geq 0\}$

ahile (b<=x) {

$\{b \leq x \wedge a^2 \leq x \wedge b = (a+1)^2 \wedge c = 2a+1 \wedge a \geq 0\}$

Aufgabe 3

$$\frac{\{P \wedge B\}\, S\, \{P\}}{\{P\} \text{ while } B \text{ do } S \{P \wedge \neg B\}}$$

$\{x \geq 0\}$

a=0

b=1

c=1

$\{a^2 \leq x \wedge b = (a+1)^2 \wedge c = 2a+1 \wedge a \geq 0\}$

while (b<=x) {

$\{b \leq x \wedge a^2 \leq x \wedge b = (a+1)^2 \wedge c = 2a+1 \wedge a \geq 0\}$

a=a+1

$\{b \leq x \wedge (a-1)^2 \leq x \wedge b = (a)^2 \wedge c = 2a-1 \wedge a-1 \geq 0\} \equiv$

$\{b \leq x \wedge (a-1)^2 \leq x \wedge b = (a)^2 \wedge c+2 = 2a+1 \wedge a-1 \geq 0\}$

c=c+2

Aufgabe 3

$$\frac{\{P \wedge B\}\, S\, \{P\}}{\{P\} \text{ while } B \text{ do } S \{P \wedge \neg B\}}$$

$\{x \geq 0\}$

a=0

b=1

c=1

$\{a^2 \leq x \wedge b = (a+1)^2 \wedge c = 2a+1 \wedge a \geq 0\}$

while (b<=x) {

$\{b \leq x \wedge a^2 \leq x \wedge b = (a+1)^2 \wedge c = 2a+1 \wedge a \geq 0\}$

a=a+1

$\{b \leq x \wedge (a-1)^2 \leq x \wedge b = (a)^2 \wedge c = 2a-1 \wedge a-1 \geq 0\} \equiv$

$\{b \leq x \wedge (a-1)^2 \leq x \wedge b = (a)^2 \wedge c+2 = 2a+1 \wedge a-1 \geq 0\}$

c=c+2

$\{b \leq x \wedge (a-1)^2 \leq x \wedge b = (a)^2 \wedge c = 2a+1 \wedge a-1 \geq 0\}$

$$\left\{a^2 \le x \wedge b = (a+1)^2 \wedge c = 2a+1 \wedge a \ge 0\right\}$$

while (b<=x) {

$$\left\{b \le x \wedge a^2 \le x \wedge b = (a+1)^2 \wedge c = 2a+1 \wedge a \ge 0\right\}$$

a=a+1

$$\left\{b \le x \wedge (a-1)^2 \le x \wedge b = (a)^2 \wedge c = 2a-1 \wedge a-1 \ge 0\right\} \equiv$$

$$\left\{b \le x \wedge (a-1)^2 \le x \wedge b = (a)^2 \wedge c+2 = 2a+1 \wedge a-1 \ge 0\right\}$$

c=c+2

$$\left\{b \le x \wedge (a-1)^2 \le x \wedge b = (a)^2 \wedge c = 2a+1 \wedge a-1 \ge 0\right\}$$

b=b+c

$$\left\{a^2 \le x \wedge b = (a+1)^2 \wedge c = 2a+1 \wedge a \ge 0\right\}$$

while (b<=x) {

$$\left\{b \le x \wedge a^2 \le x \wedge b = (a+1)^2 \wedge c = 2a+1 \wedge a \ge 0\right\}$$

a=a+1

$$\left\{b \le x \wedge (a-1)^2 \le x \wedge b = (a)^2 \wedge c = 2a-1 \wedge a-1 \ge 0\right\} \equiv$$

$$\left\{b \le x \wedge (a-1)^2 \le x \wedge b = (a)^2 \wedge c+2 = 2a+1 \wedge a-1 \ge 0\right\}$$

c=c+2

$$\left\{b \le x \wedge (a-1)^2 \le x \wedge b = (a)^2 \wedge c = 2a+1 \wedge a-1 \ge 0\right\}$$

$$\left\{b+c \le x+c \wedge (a-1)^2 \le x \wedge b+c = (a)^2 + c \wedge c = 2a+1 \wedge a-1 \ge 0\right\}$$

b=b+c

$$\left\{a^2 \le x \wedge b = (a+1)^2 \wedge c = 2a+1 \wedge a \ge 0\right\}$$

while (b<=x) {

$$\left\{b \le x \wedge a^2 \le x \wedge b = (a+1)^2 \wedge c = 2a+1 \wedge a \ge 0\right\}$$

a=a+1

$$\left\{b \le x \wedge (a-1)^2 \le x \wedge b = (a)^2 \wedge c = 2a-1 \wedge a-1 \ge 0\right\} \equiv$$

$$\left\{b \le x \wedge (a-1)^2 \le x \wedge b = (a)^2 \wedge c+2 = 2a+1 \wedge a-1 \ge 0\right\}$$

c=c+2

$$\left\{b \le x \wedge (a-1)^2 \le x \wedge b = (a)^2 \wedge c = 2a+1 \wedge a-1 \ge 0\right\}$$

$$\left\{b+c \le x+c \wedge (a-1)^2 \le x \wedge b+c = (a)^2 \wedge c = 2a+1 \wedge a-1 \ge 0\right\}$$

b=b+c

$$\left\{b \le x+c \wedge (a-1)^2 \le x \wedge b = (a)^2 + c \wedge c = 2a+1 \wedge a-1 \ge 0\right\} \equiv$$

$$\left\{b \le x+c \wedge (a-1)^2 \le x \wedge b = (a)^2 + 2a+1 \wedge c = 2a+1 \wedge a-1 \ge 0\right\}$$

b=b+c

$$\left\{b \le x + c \wedge (a-1)^2 \le x \wedge b = (a)^2 + c \wedge c = 2a+1 \wedge a - 1 \ge 0\right\} \equiv$$

$$\left\{b \le x + c \wedge (a-1)^2 \le x \wedge b = (a)^2 + 2a + 1 \wedge c = 2a+1 \wedge a - 1 \ge 0\right\} \equiv$$

$$\left\{b \le x + c \wedge (a-1)^2 \le x \wedge b = (a+1)^2 \wedge c = 2a+1 \wedge a - 1 \ge 0\right\} \equiv$$

$$\left\{(a+1)^2 \le x + 2a + 1 \wedge (a-1)^2 \le x \wedge b = (a+1)^2 \wedge c = 2a+1 \wedge a - 1 \ge 0\right\} \equiv$$

$$\left\{a^2 \le x \wedge (a-1)^2 \le x \wedge b = (a+1)^2 \wedge c = 2a+1 \wedge a - 1 \ge 0\right\} \Rightarrow P$$

Zusammenfassung

$$\left\{a^2 \le x \wedge b = (a+1)^2 \wedge c = 2a+1 \wedge a \ge 0\right\} \equiv P$$

while (b<=x)

$$\left\{b \le x \wedge a^2 \le x \wedge b = (a+1)^2 \wedge c = 2a+1 \wedge a \ge 0\right\} \equiv P \wedge B$$

a=a+1

c=c+2

b=b+c

$$\left\{a^2 \le x \wedge b = (a+1)^2 \wedge c = 2a+1 \wedge a \ge 0\right\} \equiv P$$

$$P \wedge \neg B \equiv \left\{a^2 \le x \wedge b = (a+1)^2 \wedge c = 2a+1 \wedge a \ge 0 \wedge b > x\right\}$$

$$\equiv \left\{a^2 \le x < b \wedge b = (a+1)^2 \wedge c = 2a+1 \wedge a \ge 0\right\}$$

$$\equiv \left\{a^2 \le x < (a+1)^2 \wedge c = 2a+1 \wedge a \ge 0\right\}$$

$$\Rightarrow \left\{a^2 \le x < (a+1)^2\right\}$$